Representing Proportional Relationships

1 GETTING THE IDEA

In a proportional relationship, two quantities vary directly with each other.

You can represent proportional relationships in several ways:

- an equation in the form $y = kx$, where k is the constant of proportionality or unit rate

- a graph of a straight line that passes through the origin

- a set or table of ordered pairs that show equivalent ratios

- a verbal description that describes a directly proportional relationship between two quantities

Example 1

A car technician took a car on four test drives. For each drive, the technician recorded the miles driven and the amount of gas used.

Gallons of Gas Used	Distance Driven (in miles)
4	100
2	50
1	25
0.5	12

Do the data in the table represent a proportional relationship?

Strategy Compare the relationship between data pairs.

Step 1   Write and simplify a ratio for each data pair.

$$\frac{100 \text{ miles}}{4 \text{ gallons}} = 25 \text{ miles per gallon}$$

$$\frac{50 \text{ miles}}{2 \text{ gallons}} = 25 \text{ miles per gallon}$$

$$\frac{25 \text{ miles}}{1 \text{ gallon}} = 25 \text{ miles per gallon}$$

$$\frac{12 \text{ miles}}{0.5 \text{ gallon}} = 24 \text{ miles per gallon}$$

The ratios are the same except for the last data pair.

Step 2   Do the data in the table represent a proportional relationship?

No, it is true that $\frac{100}{4} = \frac{50}{2} = \frac{25}{1}$, but $\frac{12}{0.5} \neq \frac{25}{1}$.

All the data values in the table need to be proportional in order for the table to represent a proportional relationship.

Solution   The data do not represent a proportional relationship.

Since the equation $y = kx$ represents a directly proportional relationship, $k = \frac{y}{x}$.

Example 2

Do the points on the graph represent a proportional relationship?
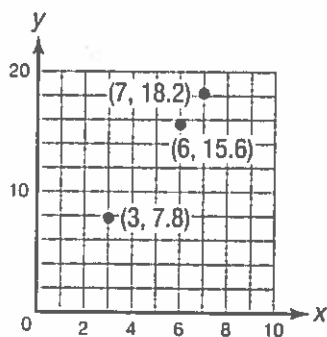
Strategy   Find the values of the ratio $\frac{y}{x}$.

Step 1   For each ordered pair, find the value of the ratio $\frac{y}{x}$.

For $(3, 7.8)$, the value of $\frac{y}{x}$ is: $\frac{7.8}{3} = 2.6$.

For $(6, 15.6)$, the value of $\frac{y}{x}$ is: $\frac{15.6}{6} = 2.6$.

For $(7, 18.2)$, the value of $\frac{y}{x}$ is: $\frac{18.2}{7} = 2.6$.

$\frac{7.8}{3} = \frac{15.6}{6} = \frac{18.2}{7} = 2.6$, so these three points are related by the same constant of proportionality, 2.6. The ordered pairs represent a proportional relationship.

Step 2   Use a graph to verify your answer.

One way to check the answer is to try to draw a straight line through all three points. If you can draw a straight line through all three points and the line passes through the origin, (0, 0), the ordered pairs represent a proportional relationship.
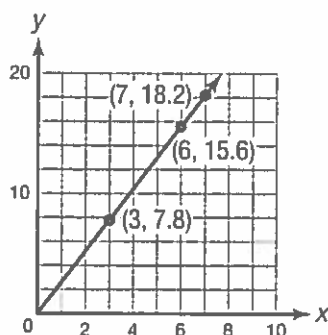
Solution   The points on the graph represent a proportional relationship.

Example 3

Show that the ordered pairs in the table represent a proportional relationship. Then write an equation to represent the proportional relationship.

x	2.5	3.0	3.5	4.0
y	7.0	8.4	9.8	11.2

Strategy   Find the ratio of y to x. Then use it to write an equation.

Step 1   Calculate the value of $\frac{y}{x}$ for any ordered pair.

For (2.5, 7.0), the value of $\frac{y}{x}$ is $\frac{7.0}{2.5} = 2.8$.

For (3.0, 8.4), the value of $\frac{y}{x}$ is $\frac{8.4}{3.0} = 2.8$.

For (3.5, 9.8), the value of $\frac{y}{x}$ is $\frac{9.8}{3.5} = 2.8$.

For (4.0, 11.2), the value of $\frac{y}{x}$ is $\frac{11.2}{4.0} = 2.8$.

Step 2   Compare ratios.

For each ordered pair, $\frac{y}{x} = 2.8$.

Since $k = \frac{y}{x}$, you know that the constant of proportionality, k, is 2.8.

Step 3   Substitute 2.8 for k in the equation for a proportional relationship.

$y = kx$

$y = 2.8x$

Solution   The ordered pairs represent a proportional relationship, where 2.8 is the constant of proportionality. The equation for the proportional relationship is $y = 2.8x$.

If the relationship between two quantities is a proportional relationship, this relationship can be represented by the graph of a straight line through the origin with a slope equal to the unit rate. For each point (x, y) on the graph, $\frac{y}{x}$ is equal to k, where k is the unit rate. The point $(1, k)$ is a point on the graph.

Example 4

The graph below shows the costs of buying pears at a farmstand. What does the point $(1, 2)$ on the graph represent? How could you use that point to help you write an equation to represent this proportional relationship?

Pear Prices

Total Price (in dollars)

Weight of Pears (in pounds)
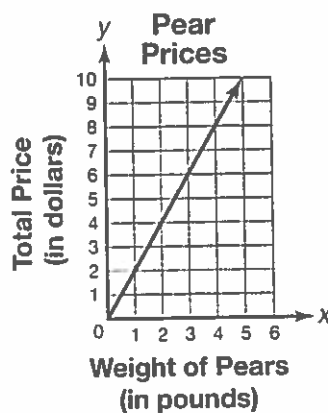
Strategy Examine the graph.

Step 1 Use the graph to examine the proportional relationship.

The horizontal axis shows the number of pounds of pears purchased.

The vertical axis shows the total price in dollars.

So, $(1, 2)$ shows that the charge for buying 1 pound of pears is $2.

Since the x-value is 1, the ratio $\frac{y}{x}$ will be a unit rate. The y-value is the unit rate.

This is the unit rate: $\frac{\$2}{1\,lb}$ or $2 per pound.

Step 2 Write an equation to represent the relationship.

The unit rate, $\frac{2}{1}$ or 2, is also the constant of proportionality. Substitute 2 for k:

$y = kx$

$y = 2x$

Solution The point $(1, 2)$ shows the unit rate—$2 per pound of pears. The equation $y = 2x$ can be used to find the total price in dollars, y, for buying x pounds of pears.

Example 5

A scientist measured the total distance, in feet, a grasshopper traveled in 3 jumps. Is the relationship between jumps and distance a proportional relationship? If so, what is the grasshopper's unit rate?

Jumps (x)	0	1	2	3
Distance in Feet (y)	0	1.5	3	4.5

Strategy Use a graph to determine if the relationship is proportional.

Step 1 Graph the ordered pairs. Then draw a line through the points.
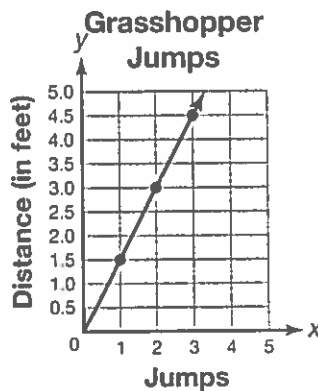
The points form a line that passes through the origin, (0, 0).

A line that represents a proportional relationship passes through the origin, so these data are directly proportional.

Step 2 Use the graph or table to find the grasshopper's unit rate.

When $x = 1$, $y = 1.5$.

The y value paired with x when $x = 1$ is the constant of proportionality, or unit rate.

The distance traveled by the grasshopper is 1.5 feet per jump.

Solution The data show a proportional relationship. The grasshopper's unit rate is 1.5 feet per jump.

Amit typed two reports for homework. He typed 240 words for the first report in 4 minutes. He typed 270 words for the second report in 4.5 minutes. Plot ordered pairs for the information given and create a graph to determine if the number of words Amit typed is directly proportional to the time it took to type the reports. Then label two additional points on the graph and describe what the points represent.

Amit's Typed Reports
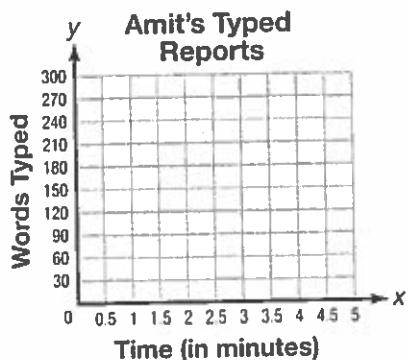
Write the ordered pairs for the data in the table.

Time (in minutes), x	Words Typed, y
4	
	270

Plot the two points on the graph. Then draw a straight line through the points. If the line passes through

_____, the relationship is _____.

Do the ordered pairs in the table represent a proportional relationship? _____

Plot two additional ordered pairs on the graph, including the point with an x-value of 1.

    Choose a point on the graph.

    The value of y is _____ when x = 1.

    Write the x- and y-value as an ordered pair: (☐ , ☐).

    This ordered pair represents Amit typing _____ words in _____ minute.

Choose another point on the graph.

    The value of y is _____ when x = _____.

    Write the x- and y-value as an ordered pair: (☐ , ☐).

    This ordered pair represents Amit typing _____ words in _____ minute(s).

Since the points _____, the data represent a

proportional relationship. The point at (1, ☐) represents Amit typing _____ words in

1 minute, and the point at (☐ , ☐) represents _____

1. Two student groups are working on the school yearbook. They printed two batches of pages on a printer in the computer lab. The table below shows the number of pages printed by each group and the time needed to print the pages.

Time (in seconds), x	Number of Pages, y
30	150
25	130

Which statement describes the relationship between the data?

A. The data represent a proportional relationship, and the constant of proportionality is 5.

B. The data represent a proportional relationship, and the constant of proportionality is 5.25.

C. The data represent a proportional relationship, and the constant of proportionality is 5.5.

D. The data do not represent a proportional relationship.

2. The constant of proportionality for a proportional relationship is k. If the value of y is k, what is the value of x?

A. 0

B. $\frac{1}{k}$

C. 1

D. k

3. The constant of proportionality for a proportional relationship is $\frac{1}{2}$. Which point does a line representing the relationship pass through?

A. $\left(0, \frac{1}{2}\right)$

B. $(1, 2)$

C. $\left(\frac{1}{2}, 1\right)$

D. $(2, 1)$

4. The total cost y is proportional to the number of used books x purchased at a constant price of \$4. Which equation represents this relationship?

A. $y = x + 4$

B. $y = 4x$

C. $y = \frac{1}{4}x$

D. $y = 4x + 4$

5. Three of the four points, $(30, 6)$, $(48, 8)$, $(25, 5)$, and $(60, 12)$, represent the same proportional relationship. Which point does **not** represent this relationship?

A. $(25, 5)$

B. $(30, 6)$

C. $(48, 8)$

D. $(60, 12)$

Use the information and graph for questions 6–9.

The graph below shows the relationship between the number of gallons of ethanol-based gasoline and the number of gallons of regular gasoline sold at a gas station.

Gasoline Sales
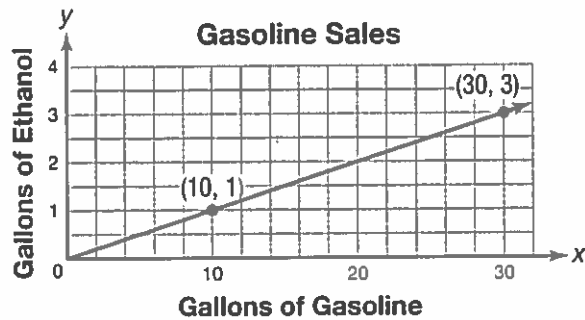
6. What is the unit rate for this proportional relationship?

   A. 0.1 gallon of gasoline per gallon of ethanol

   B. 0.1 gallon of ethanol per gallon of gasoline

   C. 10 gallons of gasoline per gallon of ethanol

   D. 10 gallons of ethanol per gallon of gasoline

7. Which equation represents the proportional relationship?

   A. $y = \frac{1}{30}x$

   B. $y = \frac{1}{10}x$

   C. $y = 10x$

   D. $y = 30x$

8. What does the point (30, 3) represent?

   A. The cost for 30 gallons of gasoline is $3.

   B. For every 30 gallons of gasoline sold, 3 gallons of ethanol is sold.

   C. The cost for 3 gallons of gasoline is $30.

   D. For every 30 gallons of ethanol sold, 3 gallons of gasoline is sold.

9. Which point is a point on the graph?

   A. (2, 20)

   B. $\left(\frac{1}{20}, 20\right)$

   C. (200, 2)

   D. $\left(2, \frac{1}{5}\right)$

Use the information for questions 10–12.

Jess says that two of the four points below represent points on the same proportional relationship:

(3, 8), (6, 15), (8, 21), and (9, 24)

10 Which two points could represent the same proportional relationship?

A. (3, 8) and (6, 15)

B. (6, 15) and (8, 21)

C. (3, 8) and (9, 24)

D. (8, 21) and (9, 24)

11 What is the constant of proportionality?

A. $\frac{3}{8}$

B. $\frac{2}{5}$

C. $\frac{5}{2}$

D. $\frac{8}{3}$

12 Which equation represents the proportional relationship?

A. $y = \frac{8}{3}x$

B. $y = 3x$

C. $y = \frac{5}{2}x$

D. $y = \frac{3}{8}x$

13 The table below shows the price of buying apples at the market.

Apples (in pounds), x	Total Price (in dollars), y
2	5
4	10
8	15

Which equation represents this relationship?

A. $y = 2.5x$

B. $x = 2.5y$

C. $y = 5x$

D. $x = 5y$

14 The ordered pair (8, 5) represents a point on the graph of a proportional relationship. Which ordered pair also represents a point on the same graph?

A. $\left(\frac{5}{8}, 1\right)$

B. $\left(1, \frac{5}{8}\right)$

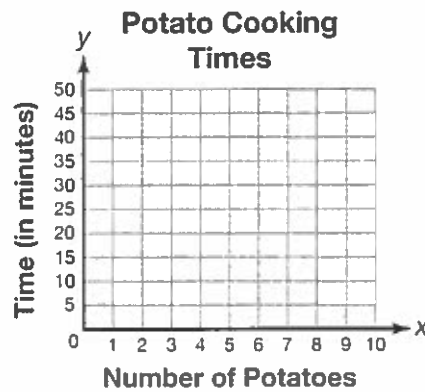C. (5, 8)

D. $\left(1, \frac{8}{5}\right)$

15 Brenda recorded the amount of time it took to cook potatoes in a microwave.

Potato Cooking Times

Number of Potatoes	Time (in minutes)
2	10
3	15
5	25

Part A

Make a graph of Brenda's data. Are the data proportional? Explain how you know.

Potato Cooking Times

Part B

Plot the point on the graph where $x = 1$. Describe what this point represents.