

6.6 Radical Equations

Obj: To solve radical equations.

To solve a radical equation:

- Get the radical alone.
- Square (or cube ...) both sides.
- Solve the resulting equation.
- You must check your answers.
- Sometimes you will get extraneous roots.
- Some problems are linear, some are quadratic.
- If it is quadratic, set it equal to zero and factor.

Examples:

$$(\sqrt{x} = 7)^2$$

$$x = 49$$

$$(\sqrt{x-6} = 8)^2$$

$$x-6 = 64$$

$$+6$$

$$x = 70$$

Examples:

$$2\sqrt{x+3} - 5 = 7$$

$$+5 \quad +5$$

$$2\sqrt{x+3} = 12$$

$$\div 2$$

$$(\sqrt{x+3} = 6)^2$$

$$x+3 = 36$$

$$-3$$

$$x = 33$$

$$(\sqrt[3]{x-1} = 5)^3$$

$$x-1 = 125$$

$$+1$$

$$x = 126$$

Examples:

$$\begin{aligned} & (\sqrt{x+5} = x+3)^2 \\ & \quad \downarrow \\ & \quad (x+3)(x+3) \\ & \begin{array}{r} x+5 = x^2+6x+9 \\ -x-5 \quad \quad -x-5 \\ \hline 0 = x^2+5x+4 \\ 0 = (x+1)(x+4) \end{array} \end{aligned}$$

Check by substitution

$$\begin{aligned} x &= -1 \\ \sqrt{-1+5} &= -1+3 \\ \sqrt{4} &= 2 \\ 2 &= 2 \star \end{aligned}$$

$$\begin{aligned} x &= -4 \\ \sqrt{-4+5} &= -4+3 \\ \sqrt{1} &= -1 \\ 1 &\neq -1 \end{aligned}$$

$$x = \{-1, \cancel{4}\}$$

NOT A SOLUTION

$$\begin{aligned} & (x-4 = \sqrt{2x})^2 \\ & \quad \downarrow \\ & \quad (x-4)(x-4) \\ & \begin{array}{r} x^2-8x+16 = 2x \\ -2x \\ \hline x^2-10x+16 = 0 \\ (x-8)(x-2) = 0 \end{array} \end{aligned}$$

$$\begin{aligned} x &= 8 & x &= 2 \\ 8-4 &= \sqrt{16} & 2-4 &= \sqrt{4} \\ 4 &= 4 \star & -2 &\neq 2 \end{aligned}$$

$$x = \{8, \cancel{2}\}$$

Examples:

$$\begin{aligned} 5 - \sqrt[4]{x} &= 0 \\ +\sqrt[4]{x} & \quad +\sqrt[4]{x} \\ \hline (5 = \sqrt[4]{x})^4 \\ 625 &= x \end{aligned}$$

$$\begin{aligned} \sqrt{2x+8} - 4 &= 6 \\ +4 & \\ \hline (\sqrt{2x+8} = 10)^2 \\ 2x+8 &= 100 \\ -8 & \\ \hline 2x &= 92 \\ \div 2 & \\ x &= 46 \end{aligned}$$

Examples:

$$\begin{array}{r} \sqrt{3x+2} - 2\sqrt{x} = 0 \\ \phantom{\sqrt{3x+2}} \phantom{-2\sqrt{x}} + 2\sqrt{x} \\ \hline (\sqrt{3x+2} = 2\sqrt{x})^2 \end{array}$$

$$3x+2 = 4x$$

$$\begin{array}{r} 3x+2 = 4x \\ - 3x \\ \hline 2 = x \end{array}$$

$$\begin{array}{r} \sqrt{4x+28} - 3\sqrt{2x} = 0 \\ \phantom{\sqrt{4x+28}} \phantom{-3\sqrt{2x}} + 3\sqrt{2x} \\ \hline (\sqrt{4x+28} = 3\sqrt{2x})^2 \end{array}$$

$$4x+28 = 9(2x)$$

$$\begin{array}{r} 4x+28 = 18x \\ - 4x \\ \hline 28 = 14x \end{array}$$

$$\frac{28}{14} = \frac{14x}{14}$$

$$\frac{28}{14} = x$$